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ABSTRACT

Recent emphasis on discourse in mathematics classrooms has spurred a line of inquiry about different forms of talk in these settings. If mathematical thinking is understood to be a set of practices that includes mathematical discourse, argumentation, which has an especially important role in mathematics, requires analytic attention. In particular, the following questions arise: How can classroom discourse be organized to support mathematical disagreements that (a) are intellectually productive, and (b) minimize social discomfort? This paper investigates the interactional organization of public disagreements in Deborah Ball's third grade classroom by describing a participant structure called accountable argumentation. The norms, expectations, interactional roles, and use of history employed during accountable argumentation are explicated and then applied to the analysis of two public peer disagreement episodes that take place during the same whole-class discussion. These two episodes illustrate the ways in which accountable argumentation supports mathematical learning through disagreement, while mitigating the potentially uncomfortable feelings typically expected in such interactions. (Contains 35 references.) (Author/ASK)

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ACCOUNTABLE ARGUMENTATION AS A PARTICIPANT STRUCTURE TO SUPPORT LEARNING THROUGH DISAGREEMENT

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ABSTRACT

Recent emphasis on discourse in mathematics classrooms (e.g. NCTM, 1991) has spurred a line of inquiry about different forms of talk in these settings. If mathematical thinking is understood to be a set of practices that include mathematical discourse, argumentation, which has an especially important role in mathematics, requires analytic attention. In particular, the following questions arise: How can classroom discourse be organized to support mathematical disagreements that (a) are intellectually productive, and (b) minimize social discomfort? This paper investigates the interactional organization of public disagreements in Deborah Ball's third grade classroom by describing a participant structure called *accountable argumentation*. The norms, expectations, interactional roles, and use of history employed during accountable argumentation are explicated and then applied to the analysis of two public peer disagreement episodes that take place during the same whole-class discussion. These two episodes illustrate the ways in which accountable argumentation supports mathematical learning through disagreement, while mitigating the potentially uncomfortable feelings typically expected in such interactions.

INTRODUCTION AND OVERVIEW

Classroom discourse practices have become an important site of inquiry and change in mathematics education (Lampert & Blunk, 1998; NCTM, 1991). As students endeavor to make sense of mathematics, their talk and interaction in the classroom provides an important resource for the construction of meaning (O'Connor, 1998). Classroom discourse comprises an important part of the "social plane" (Vygotsky, 1981) in which higher order thinking first appears before it becomes appropriated by individuals.

Argumentation has an especially important role in mathematical thinking (Lakatos, 1976). If students are to engage in authentic mathematical activity while making sense of the subject, then argumentation and disagreements should find a way into the discursive practices of mathematics classrooms. In discussion-intensive classrooms, students can be, in a sense, apprenticed to these argumentation and reasoning practices through their participation in the classroom (Lave & Wenger, 1991).

However, as teachers of discussion-intensive classrooms relate (e.g. Lampert et al., 1994), these disagreements come with the social risk of "losing face" in front of peers. A question thus arises: How can classroom discourse be organized to support mathematical disagreements that (a) are intellectually productive, and (b) minimize social discomfort?

A well-known videotape of Deborah Ball's third grade mathematics classroom provides ideal data for investigating this question. During the class session of January 19, 1990, Ball's students generate a "new" class of numbers while arguing vigorously about the nature of odd and even numbers. The class session is thus an exemplary case of mathematical argumentation that is both intellectually productive and socially viable.

The videotape of the class session starts with a discussion of the previous day's meeting with the fourth graders, in which the evenness or oddness of the number zero was the topic of debate. During the course of the class discussion, a student named Sean[1] proposes that the number six can be both even and odd. The class pursues this idea, with many students stepping in to disagree with Sean's statement. Over the course of the class session, the proposal about six eventually becomes generalized as an example of the

[1] The students' names are pseudonyms.

category of "odd-and-even" numbers. (As it turns out, the "odd-and-evens" — or, as they came to be known, "Sean numbers" — are the even numbers which have an odd number of groups of two, e.g. 2, 6, 10, . . .)

This paper will focus on describing the interactional organization of *accountable argumentation*, an activity embedded in whole-class discussions in Ball's classroom. An analysis of the classroom's interactional organization will provide insight into how this particular discussion managed to be both intellectually productive and socially viable.

Organization of paper

This paper is organized in the following manner. First, I locate this analysis theoretically, tracing its connections to analyses of both classroom participant structures and the discourse of disagreements. In the second part of the paper, I explicate the classroom's interactional organization by describing a participant structure, which I call *accountable argumentation*, which supports mathematical learning through disagreement. I then provide the analyses of the two disagreement episodes described above. Finally, I conclude with a discussion of accountable argumentation as a pedagogical and analytical resource for investigating mathematical learning.

THEORETICAL PERSPECTIVES

Participant structures

To understand how the class coconstructed the odd-and-even numbers during the class session, it is helpful to examine the classroom discourse practices. Classroom discourse practices are socially developed, patterned ways of using language, gesture, and representations, coordinated with understandings about the subject matter (Greeno et al, 1998). One analytic construct used to understand these patterned interactions is that of classroom *participant structures* (Philips, 1983/1993). Participant structures are interactionally emergent, providing and organizing resources for learning (Hanks, 1991). Participant structures allocate student involvement in classroom activities and produce a level of interactional organization within which the structuring of any single encounter is accomplished (Philips, 1983/1993, p. 79). The "structure" is, however, a highly contingent one; the "rules" provide resources for participants to play the game according to their

own strategies and are not simply obeyed (Lemke, 1993, p. 9). Some examples of participant structures commonly found in classrooms are *whole class discussion*, *group work*, and *teacher tutoring*.

On a descriptive level, these participant structures provide a way of seeing a given activity. On an analytic level, participant structures allow for the investigation of the relations between social actors and their interactions. That is, the examination of classroom participation structures supports the analysis of the relations between students, the teacher, and the various classroom activities, providing insights into the organizational possibilities and limitations for participation and learning. In studying participant structures, researchers seek to codify (a) these relations, (b) the corresponding positions, and (c) the normative expectations for appropriate conduct. From there, interaction between participants can be highlighted and analyzed (Goffman, 1981; Hanks, 1996). In educational research, analyses of the organization of classroom participation — and the way various classroom participants draw on, resist, and transform that organization — allow for context-specific descriptions of learning.

The analysis of discourse and interaction for understanding mathematics and science learning is a recent and evolving project which has its precedents in the work of other educational researchers, including Hall and Rubin (1998), John (1997), Lemke (1993), O'Connor and Michaels (1996), and Stevens and Hall (1998). This paper seeks to continue this line of work by explicating a participant structure called *accountable argumentation* that organizes public disagreements in the context of whole-class discussions in a mathematics classroom and supports the development of mathematical ideas.

The discourse of disagreements

To understand disagreements in this classroom, I will draw on three sources. The first source is Ball's frequent collaborator,[2] Magdalene Lampert, who, along with her colleagues, has written about the role of disagreements in her classroom (Lampert, 1998; Lampert, Rittenhouse, & Crumbaugh, 1996). Lampert et al.'s analysis describes the social and personal tensions that arise as students engage in academic disputes, which the authors see as having a place in mathematical learning. They argue that engaging in disagreement is closer

[2] See, for example, Lampert and Ball (1998) for a description of their mutual work.

to authentic mathematical and scientific practice than the types of discourse that typify traditional classrooms. As the authors say, "by posing interpretable problems and encouraging disagreement, the teacher *sets the stage* for students to clarify their thinking and relate thought to communication" (p. 738; italics added). How that stage is set, the roles available to the players, and the discursive details of the drama of disagreements are not a focal part of their analysis but are explored in this paper. Interestingly, in those discursive details lie a multitude of interactional strategies that participants employ to manage the social and personal tensions as they engage in disagreements.

Including disagreements in the teaching of mathematics is, in part, an attempt to bring mathematical pedagogy closer to mathematical practice (Chazan & Ball, 1995; Lampert et al, 1994). Accounts of authentic scientific practice thus play an important role in sorting out some of the complexities that arise during academic disagreements. Bruno Latour's (1987) work on this aspect of scientific practice provides a way to delve more deeply into these complexities. Specifically, his analysis provides a language for the strategies scientists use (e.g. recruiting allies, reifying ideas through inscriptions) and the positions they take (e.g. dissenter) in the course of disagreements. It turns out that the members of Ball's classroom, in their appropriation of scientific argumentation, employ some of the same strategies and position themselves in ways similar to the professional scientists.

Finally, because the students are children who bring in their own personal understandings of how disagreements are managed, Marjorie Goodwin's (1991) linguistic ethnography of urban black children's talk provides insight into the discourse of peer disputes. Her fine-grained ethnographic work describes many of the relational stakes involved in such arguments, as well as a detailed analysis of the strategies children employ in these interactions, without the mediation of adults.

Bringing these three perspectives together, this paper will elaborate the role of language as a resource for positioning students in the course of mathematical disagreements in the classroom. Specifically, this analysis will illustrate the participant structure of accountable argumentation. *Accountable argumentation* is a participant structure embedded in whole class discussion that organizes the public disagreements between students and provides interactional resources for clear mathematical reasoning and the production of mathematical generalizations. As will be elucidated,

accountable argumentation is organizationally distinct from whole class discussions because, although it is comprised of public talk, (a) turn-taking is managed by the students engaged in accountable argumentation, not by the teacher; (b) it places students at the interactional and often physical center of the classroom; and (c) it permits students freer movement and greater access to classroom resources such as the chalkboard. Accountable argumentation also provides interactional roles to students to support their mathematical positioning during disagreements, as well as communicating and supporting the expectation that students justify these positions to their classmates.

DATA AND METHODS

Data

Deborah Ball provided access to her rich set of data from this classroom, which was collected as part of the Mathematics and Teaching Through Hypermedia (M.A.T.H.) Project. These data include: the videotape of the January 19, 1990 class session; a transcript of the session; a transcript of the prior day's class session; her journals from the days leading up to, including, and after January 19; copies of the students notebooks from January 19; observation notes from January 18, 19, and 23; and prior analysis of the day's events (Ball, 1998).

Methods

Methods from both conversation analysis and sociolinguistics were employed in this analysis. Conversation analysis seeks to describe the underlying organization of social interaction. To do so, it requires an integrated analysis of action and local context, with the assumption that speakers constantly influence and constrain the conduct of their coparticipants. Because coparticipants in interaction rely on spoken utterances or other actions to interpret a situation, this, too, is the point of departure for conversation analysts (C. Goodwin & Heritage, 1990; Sacks, Schegloff, and Jefferson, 1974). The utterances and actions of the students and teacher comprised the primary data on which this analysis relies.

Sociolinguistics adds depth to the conversation analytic notion of context by using ethnographic methods to interpret interactions beyond the immediate and local context created by talk and action alone (M. Goodwin, 1991; O'Connor & Michaels, 1996). Thus, the analysis in this paper seeks to

interpret the spoken utterances and gestures of the class participants in the broader contexts of interaction, such as the previous day's class and the teacher's perspective as represented by her reflections on the class discussion. Although the interpretations of the context did not come from traditional ethnographic data, the rich and various data that Ball's group provided deepened the understandings of the multiple contexts that informed the local interactions.

As a consequence of the close attention paid to language and action in this analysis, the data provided required some transformation to lend themselves to fine-grained analysis of interaction. In particular, portions of the transcript of the January 19, 1990 class session were revised to include more of the false starts, repeated words, and acknowledgment tokens ("mm hm"'s), using transcript conventions common to this type of analysis (Ochs, 1979; see Table 1 below). These additional utterances, along with notes about gesture, physical location, and intonation, become important clues for understanding and interpreting interaction in context. The trade-off, of course, is that the transcripts' readability may be diminished, but for those accustomed to the notation, these transcripts provide greater access to the sound and pace of the conversations.

[Insert Table 1 here]

The revised transcripts were analyzed inductively (Glaser & Strauss, 1967), along with the videotape and supporting artifacts from the January 19 class session. From this analysis, the participant structure of accountable argumentation emerged as a significant organizer of interaction during whole class discussion. Its components will be explicated in the next section of the paper.

ANALYSIS: ACCOUNTABLE ARGUMENTATION IN ACTION

Overview of analysis

The following analysis provides an abstract and then an enacted description of accountable argumentation. First, I describe the abstract structure of accountable argumentation by delineating the norms, expectations,[3] roles,

[3] Of course, since accountable argumentation is embedded in whole-class discussions, the two participant structures have some norms and expectations in common.

and uses of history that define it. Then, I illustrate accountable argumentation by analyzing two disagreement episodes. Episode 1 occurs during the first six minutes of the January 19, 1990 class session and illustrates how accountable argumentation, compared to other participant structures in the classroom, supports mathematical reasoning and learning. I then analyze a second episode, a disagreement that occurs later in the class session, to show how the interactional roles in accountable argumentation provide a resource for the development of mathematical ideas. During Episode 2, the odd-and-even concept is made more general, and therefore more mathematical.

Accountable argumentation : The abstract structure of participation

Norms

The norms of this classroom are constantly enacted through participants' interactions. These contribute to the organization of participants' activities by both constraining and providing a resource for these activities. Since accountable argumentation is embedded in the whole class discussions in Ball's classroom, many of the norms described are common to both participant structures.

Listed below are some norms that structurally distinguish accountable argumentation from what might be thought of as more typical whole-class discussion. Each of these norms is briefly described.

1. **Accountable argumentation uses terms from the mathematical and academic registers (e.g. "proof," "conjecture").**
   During disagreements, participants use words from the mathematical register, including "proof," "conjecture," and "definition." They also use other academic terms to describe their thinking, such as "revise." This reflects the ways in which accountable argumentation appropriates authentic mathematical and academic practices by legitimating the thinking and reasoning activities described by these words. By having students engage in and name such activities, these registers also lend the disagreements a distinctly academic character, perhaps helping to alleviate some of the potentially personal feelings of engaging in conflict.
2. **Discussions have a slow and measured pace.**

As Rowe's (1986) work on wait time has shown, teachers typically do not wait for more than one second for students to respond in conversation. In contrast, during the conversations of January 19, 1990, students commonly take significant pauses as they are thinking through their ideas — even pauses as long as seven seconds. This normative pace provides interactional support for participants to think through and substantiate their positions.

3. **Disagreements are important and may not (and need not) be resolved.** As will be illustrated in the analysis of Episode 1, disagreements in Ball's classroom differ from everyday disagreements. First, they are not avoided or negatively evaluated, and, second, they do not require a resolution.

Together, these norms legitimate and support reasoning activities in Ball's classroom. Language and time are provided for thinking activities (norms 1 and 2), and the common discomforts associated with everyday disagreements are alleviated by the normalization and the containment in academic language of these disputes (norms 1 and 3).

Expectations

Accountable argumentation holds participants to the expectations listed below. Again, some of these expectations are not specific to accountable argumentation and apply equally to other forms of classroom participation. The expectations listed below are constantly communicated during interactions. They are invoked, animated, or met in interaction, and are illustrated by transcript excerpts from the first six minutes of the class session. In this class, participants are expected to:

1. **Attend to contributions in a whole-class discussion**

Ball: And could you listen to one another's comments, so that we can, um, (1) benefit from what other people say?

2. **Have a justified position in a discussion**

Mei: [...] I thought that zero was always going to be a even number, but from the meeting I sort of got mix ed up, because I heard other ideas I agree with and now I don't know which one I should agree with.
Ball: Mhm. So what are you going to do about that?
Mei: U::m, I'm going to listen more to the discussion and find out.

3. **Act on or defend a position in a discussion**

Sean: [...] I disagree because, um, because what= what two things can you put together to make it?

4. **Respectfully respond to other's positions in a discussion**

Sheena: (1) Could you repeat what you said, please?

****

Sean: (3) I didn't think of that that way. Thank you for bringing it up

5. **Revise a position in light of new questions or convincing evidence.**

Ball: [...] Sheena commented that (.) it was good to have the two classes together because she heard an idea that she hadn't thought about and it made her think about and even revise her own idea when she was (.) in the meeting yesterday.

These norms and expectations are communicated consistently through the participants' interactions. At times, their violation is noted explicitly, as when a student is not listening attentively (e.g. "Were you not listening to this just now?") or when somebody has not adequately substantiated a controversial position (e.g. "Prove it to us!"). Note that the norms and expectations of accountable argumentation vary radically from those of traditional mathematics classrooms, where the authority of the text or teacher supersedes students' valuation of their own thinking (Schoenfeld, 1990).

Roles

The roles described below are those that are taken up and enacted during accountable argumentation sequences during whole-class discussions. This description of roles reflects an inductive analysis of the discursive practices in the classroom, and their labels have been borrowed from other accounts of disagreements in both discourse analysis (Goffman, 1974) and the social studies of science (Latour, 1978).

The following roles are available to participants engaged in accountable argumentation:

1. **Principal of a controversy** (Goffman, 1974, p. 517): A person held accountable for a position that others disagree with or question;
2. **Dissenter** (Latour, 1987): A person who takes an opposing position to the principal's position;
3. **Ally** (Latour, 1987): A person (or thing) who supports the principal's position;

During disagreements, participants may take up any of these first three roles. Oftentimes, they blend them with one or more of the following:

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4. **Questioner:** A person who asks questions, especially to the principal, perhaps because of confusion or an uncertain stance;
5. **Reasoner:** A person who provides an exposition of reasoning;
6. **Listener:** A person who listens to the arguments;
7. **Norm-maintainer:** A person who explicitly evokes norms during conversation;
8. **Clarifier:** A person who clarifies or summarizes another participant's statement.

In this description, I do not wish to suggest that these roles are explicitly delegated or stable over time. The roles are occupied through highly contingent, negotiated social processes which will be explicated in the following section.

Negotiating roles in interaction.

Participants in discussion can utilize these roles as resources in several ways. First, they can *assume* (Goffman, 1974) the various roles for themselves through their talk. For example, by beginning a speaking turn with the statement, "I disagree with Joe," a participant assumes the role of *dissenter*. Secondly, participants can *design* (Sacks, Schegloff, & Jefferson, 1974) others in these roles. With the same utterance ("I disagree with Joe"), the speaker designs Joe as a *principal of a controversy*. Once that role has been *ratified* (C. Goodwin & Heritage, 1990) by the designated principal — that is, Joe acknowledges this role interactionally — the stakes are then raised for Joe to act on or defend the controversial position for which he is now accountable. Finally, participants can *animate* (M. Goodwin, 1990) others in these roles. In the midst of summarizing an argument, for instance, a speaker may juxtapose the positions of two participants to point out their agreement on an issue. In this case, the two participants are animated as *allies*.

History

Through animation, speakers can strategically recycle past utterances or positions in the present interaction. By animating a past speaker's position, current speakers can recruit allies or take positions of dissent. Additionally, because participants are frequently animated as holding positions from previous class sessions, this continuity intensifies participants' day-to-day

accountability to their positions on a topic. Participants may be *yoked* (O'Connor & Michaels, 1996) into a discussion when their past positions are recycled, connecting them to present discussions through the animation of a current speaker.

Some aspects of history are also brought into the present through the use of *inscriptions* (Latour, 1987), representations which reify ideas, serve as pedagogical devices, support talk over and about them, and support the recycling of past arguments (Greeno & Hall, 1997; Roth & McGinn, 1998). Inscriptions provide important mediating resources for argumentation. In fact, inscriptions themselves are often animated as allies during reasoning sequences, as will be illustrated in the Episode 1 below.

In this classroom, a few inscriptions are featured in the course of the discussion. These are representations of numbers from which evenness or oddness may be derived, including the number line representation, a hash mark representation (for example, 4 would be represented as | | | |), and a "cookie" representation of numbers (o o o o). These representations support different understandings of even and odd. That is, the number line highlights the alternation of even and odd integers, and, although both the hash mark and cookie representations allow for "grouping by twos," individual cookies are more easily "split in half." These inscriptions and their entailments are summarized in Table 2.

[Insert Table 2 Here]

Accountable argumentation illustrated

In this section, accountable argumentation will be illustrated through the analysis of two episodes of interaction in the context of whole-class discussion. The features of the abstract structure — that is, the norms, expectations, roles, and uses of history — will provide the conceptual language for this analysis. Episode 1 will contrast accountable argumentation to other participant structures to illustrate the opportunities for learning that it provides. Episode 2 will show how the roles in accountable argumentation support novel mathematical thinking.

Episode 1: A disagreement about zero

Overview of Participation Structures in Episode 1. This episode traces a disagreement about zero as enacted by two students in Ball's class, Sean and Sheena. The episode starts in a whole class discussion format in Part 1, with

Sheena discussing with Ball her current thinking about the evenness or oddness of zero after the previous day's meeting. An accountable argumentation sequence begins in Part 2, when Sean disagrees with Sheena's position, and continues in Part 3, when Sheena defends her position. Accountable argumentation gives way to a peer dispute format (M. Goodwin, 1991) at the end of the episode, at which point Ball steps back in and redirects the activity to a whole class discussion. In addition to illustrating some important features of accountable argumentation, this episode shows the contiguous relationships of the participant structures in the classroom and the fluid transitions between them.

Part 1: Reflecting on a position. At the beginning of the class session, Ball opens up the discussion by asking that students reflect on the previous day's meeting with the fourth graders on the topic of zero's oddness or evenness. Structurally, this is a whole-class discussion format with the teacher allocating turns of talk and designating the topic, which is "reflection on yesterday's meeting." Sheena, in dialogue with Ball, goes public with the position she has arrived at:

5	Ball:	Was there an example of something yesterday that (.) you understood a little bit more (.) during the //meeting?]

6	Sheena:	//Well:],
		I didn't think that zero was (.) zero, um even or (.) odd until yesterday they said that it could be even because of the ones on each side is odd, so that couldn't be
		//odd.]

7	Ball:	/Hmm.]] So y=
8	Sheena:	=So that helped me understand it.

In this sequence of dialogue, Ball supports Sheena in taking a specific position on the topic of yesterday's discussion by asking her to provide a specific example of something she "understood a little bit more." In turn 6, Sheena recycles an argument linked to the number line inscription to substantiate her reasoning ("because the ones on each side is odd, so that couldn't be odd"), bringing part of yesterday's discussion into the present interaction. She describes being convinced by something "they said" (turn 6)

that "helped [her] understand it" (turn 8), conforming to the expectation that her position is reasoned.

Part 2: Initiating accountable argumentation. When Sean is called on as the next student to speak, he reorganizes the interaction from a discussion reflecting on yesterday's meeting to accountable argumentation:

11	Ball:	Other people's comments? Sean?
12	Sean:	Um, I, I, I just want to say something to Sheena =when sh/ what she said about um (.) that (.) that one, um zero has to be an odd/an even number bec/ I disagree because, um, because what= what two things can you put together to make it?
13	Sheena:	(1) Could you repeat what you said, please?
14	Ball:	*((speaks to Betsy and asks her to listen))*
15	Sean:	(1) Okay, um, I disagree with you because (.) um, if it was an even number, how/what two things could make it?

Perhaps because he is redirecting the activity from reflection to accountable argumentation, Sean somewhat haltingly prefaces his disagreement in turn 12. In this preface, Sean addresses Sheena indirectly, using her name ("I, I, I just want to say something to Sheena"). After Sheena directly addresses him (turn 13) and he takes a slight pause, Sean addresses her in the second person ("Okay, um, I disagree with you"). This pattern of assuming a position of dissent by starting in third person ("Sheena") and changing to second person ("you") only after the dissent has been ratified by other participants occurs elsewhere during the class session. This may mark a transition point from addressing the teacher in whole class discussion to addressing a peer in accountable argumentation. It also seems possible that this strategy helps students manage the relational discomfort of disagreeing with one another. By initially addressing the principal of the controversy in the third person, participants help manage the potentially personal feelings of disagreement.

After Sean designs Sheena as the principal of a disagreement, Sheena ratifies the role of principal and now takes on the rights and obligations of that role. Principals in this classroom, for example, are obligated to defend their position in a disagreement by explicating their reasoning. They also have the right to assume a central position in the classroom, both physically and discursively, as will be illustrated in this example (and in Figure 1 below).

In turn 15, Sean formulates a question for Sheena. He merges the roles of dissenter and questioner, which seems to be a common way for dissenters to account for their own positions in this classroom. Sean supports his dissent by asking a question that reflects his understanding of even numbers. That is, even numbers are made up of two like numbers, an understanding supported by the circle or hash mark representations. By posing a question, Sean also designs Sheena as having a reasoned position from which she can respond and underscores her obligation to defend her now-controversial position.

When students have stepped into the roles of principal and dissenter, Ball no longer manages speaking turns from her position of teacher. The student-student dialogues during whole-class discussions appear to occur during accountable argumentation sequences, once the roles of principal and dissenter have been assumed and ratified in interaction. This aspect of accountable argumentation decenters the teacher's authority to mandate turns, allowing students to manage some of their own interaction.

Embedded in this sequence of dialogue, Ball quietly works as a norm-maintainer by asking Betsy to listen to the discussion. In doing so, she designs Betsy as a listener in the discussion and underscores the expectation of attending to the arguments in a controversy, even when one does not hold the floor.

Part 3: Defending a position. Sheena then goes on to defend her position:

16 Sheena: Well, I could show you it. (.)
((Moves toward the chalkboard and points to the number line with a yard stick))
Um, I forgot wh/what his name was, but yesterday he said that this one
((Points to the 1 on the number line))
and each, this one is odd and this one
((Points to the -1 on the number line))
is odd, so this one
((Points to the zero))
has to be even.

[Insert Figure 1 about here]

Sheena, in the position of principal, must justify her position on zero. To support her effort, she moves into what is typically the teacher's position

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at the front of the room near the chalkboard, a location of interactional authority and visual centrality.

Additionally, she recruits two important allies from yesterday's meeting, appealing to the history of this controversy over zero. During the exposition of her position, Sheena first allies herself with one of the fourth grade boys ("I forgot wh/what his name was"), giving him authorship of her reasoning over the number line ("but yesterday he said"). This move accomplishes two things for Sheena. First, the alliance with an older (and perhaps therefore more authoritative) boy shows that Sheena's position is not anomalous but rather shared by another, perhaps more knowledgeable, peer. Secondly, this move alleviates some of the relational pressures of the principal position, as the anonymous boy is the author of the argument and not Sheena herself.

Sheena also invokes the number line inscription (and, as a consequence, the argument it represents) as an ally to substantiate her position, even though Sean's question actually refers to a representation of even numbers (turn 15: "what two things could make it?") that relies on the notion of even numbers being comprised of same-sized pairs. They are thus engaged in cross-representational discussion, as the alternating pattern of the number line and the "two-groups" representations reflect different aspects of evenness.

Part 4: From accountable argumentation to peer dispute.[4] At this point, perhaps because of the seemingly incommensurable representations of evenness, the discussion shifts and takes on a format sometimes found in peer disputes (M. Goodwin, 1990):

17	Sean:	But that doesn't mean it always is even.
18	Sheena:	It *could* be even.
19	Sean:	It *could* be, but
20	Sheena:	I'm not saying that it *ha::s* to be even. //I meant that it could be.]

[4] By drawing this contrast, I do not wish to suggest that peer dispute never has features of accountable argumentation. Marjorie Goodwin (1990) describes, for example, the uses of proof and justification that are often employed within peer disputes. The polite tones, public forum, classroom setting, and appropriation of the mathematical register do seem to distinguish accountable argumentation from typical peer disputes.

21 Sean: // You said it was]

((Sheena puts the yardstick back in the chalk tray.))

22 Ball: (2) Before we take this up again, I underst/I (.) I understand that this is still a problem = and that we didn't a/we didn't settle it. (.)

((Sheena returns to her seat.))

We're probably not going to settle it. (.) U:m (.) there's a lot of disagreement about this issue, right? And you saw that the fourth graders who have been thinking about this for a long time also disagree about it, don't they? I'M STILL kind of interested (.) um:, in hearing some more comments about the meeting *itself*. [...]

Sean's challenge in turn 17 recalls a category of numbers that emerged in the prior day's discussion. In conventional mathematics, even and odd are mutually exclusive categories of integers. During the previous day's meeting, some students held the position that zero is perhaps both odd-and-even, a special number that straddles categories. The "odd-and-even" category eliminates the mutual exclusivity of even numbers and odd numbers, which the number line representation reifies.

In turns 18 through 20, Sheena weakens her commitment to her position that zero is even by hedging ("It *could* be even"), further distancing herself from her original position in the face of Sean's dissent. Her position in turn 6 (zero "couldn't be odd") has been transformed in the course of this disagreement to the statement in turn 20. Sean's response (turn 19), a partial repetition of Sheena's prior talk, does not represent further reasoning about the issue at hand but rather serves mainly to escalate the dispute.[5] Sean's addition of "but" might index his previous turn 17 ("But that doesn't mean it always is even"), thus continuing to push his position without providing further justification. At this phase of the dispute, neither Sheena nor Sean formulate explanations to accompany their assertions. This "exchange and return" sequence (M. Goodwin, 1990) indicates that the students are no longer focused on the validity or invalidity of one another's statements. The peer dispute format that now organizes the exchange subsumes accountable argumentation's emphasis on reasoned positions.

Sheena eventually retreats from her original stance and replaces the yardstick in the chalk tray (turn 20), signaling the termination of the dispute,

[5] This partial repetition of a previous utterance is an example of "format tying" (M. Goodwin, 1990), a reciprocal action which transforms Sheena's meaning and escalates the dispute.

even though no explicit resolution has been reached.[6] Sheena continues to stand at the center of the room by the chalkboard in the physical position of principal, although she has discarded her primary tool. This signifies an ambiguous moment of role transition, open for design by others.

After a two second pause, Ball steps in to normalize the unresolved ending (turn 22). When she does so, Sheena picks up the cue to relinquish her role as principal and returns to her seat. Ball says aloud that "we didn't settle it," referring to the disagreement about zero. Her use of the pronoun "we" reestablishes the dispute as a collective issue for the whole class to grapple with, clearly demarcating it from a personal conflict between Sean and Sheena. Ball continues to underscore the unresolved status of this issue, emphasizing that "we're probably not going to settle it," before she redirects the activity to whole class discussion.

Discussion of Episode 1. This disagreement illustrates some of the social stakes of publicly stating one's position on an issue. Sheena, in the context of reflecting on the previous day's meeting, is encouraged by Ball (turns 5 through 8) to have a principled position about zero's oddness or evenness. In that context, she elaborates her stance to Ball in order to fulfill the expectation of supporting her position. In doing so, however, she makes her position public, which then opens her up to dissent from others. When Sean acts as a dissenter to her position, she manages the pressure of the role of principal of a controversy by recruiting allies and distancing herself from authorship of her position. The cross-representational discussion escalates the disagreement. At this point, accountable argumentation is abandoned for a peer dispute format that does not rely on reasoned positions, at which point Sheena hedges and eventually retreats from her stance.

When Ball steps in and redirects the activity to whole-class discussion, she appears to be choosing to return to the original topic and format that she set out (a whole class discussion reflecting on yesterday's meeting). Had she wished to pursue the disagreement further, accountable argumentation could have continued. Although Sean and Sheena had arrived at an impasse, it is possible that a *clarifier* or a *questioner* — be it Ball or another student — could have entered the conversation to point out or ask about the mismatch

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[6] Such closings provide what Goffman (1971, p. 140, as cited in M. Goodwin, 1990) calls "ritual equilibrium."

between their respective representations. It is important to note that it is Ball's choice to end the accountable argumentation sequence and leave the dispute unresolved. She prioritizes her other objectives for that day's lesson over the resolution of this disagreement.

Even though the disagreement ends without a visibly reasoned resolution, the structure of accountable argumentation still supports mathematical learning in this episode. Comparing, for example, Sheena's first statement in her position on zero (part 1, turn 6) to her explanation as principal (part 3, turn 16), we see that the latter explanation is more clearly stated. Standing at the center of the room, Sheena's communication has a different purpose: she is no longer communicating her reflections on yesterday's meeting to her teacher, but she must defend herself in front of the whole class because her position has been called into question. From this central location, she has the number line inscription and yard stick to use as resources to support her justification. In this example, we see how accountable argumentation supports Sheena's mathematical learning. First, by shifting the audience from a sympathetic teacher to a dissenting peer, accountable argumentation provided a compelling impetus for her to become more articulate and formal about her position. Additionally, by entitling Sheena, as principal, to the interactional center of the room with its visible representational resources, accountable argumentation provided a means for her to step into a teaching position from which she can access and display inscriptions to support her justification.

Accountable argumentation as a participant structure uniquely supports this learning. Had the class' activity retained the organization of whole class discussion, Sheena would not have necessarily needed to repeat her position or find a way to articulate it more clearly. Similarly, the linguistic and interactional resources of accountable argumentation supported the productive disagreement in Parts 2 and 3 of this episode, during which Sean states his disagreement and Sheena defends her position. In contrast, the peer dispute format in Part 4 does not support displays of reasoning by shifting the focus away from evaluating the validity of one's argument to posturing and hedging.

Episode 2: A disagreement about six

The controversy about six alluded to in the introduction of this paper consumes much of the class session. It begins when Sean publicly states that he thinks six is an odd and even number, a statement that quickly incites dissent from his peers. It is in arguing this controversy that the class participants coconstruct the odd-and-even numbers 2, 6, 10, . . . (As mentioned earlier, these numbers can be described as even numbers that have an odd number of groups of two.)

By tracing the progression of this idea, we can see how accountable argumentation provides the interactional resources for the development of the odd-and-evens. With Sean taking the role of principal in this controversy, we can see how, through their dissent, the other participants in the class collaboratively transform his utterance from a particular statement about six ("it could be an *odd* and an *even* number") to a general class of numbers. This movement from the particular to the general is a quintessentially mathematical one and, as will be argued here, is a type of transformation supported by accountable argumentation.

While many participants undoubtedly contributed to the discussion and, more specifically, to the construction of the odd-and-evens, I will focus on one interaction between Sean and Mei in which accountable argumentation supports the transformation of the statement about six.

Overview of Roles in Episode 2. While Episode 1 contrasted accountable argumentation to other participant structures in its capacity to support student learning, Episode 2 will illustrate the ways in which the roles of accountable argumentation may help support generative mathematical thinking. In Part 1 of this episode, Sean, as a principal of the controversy about six, is struggling to articulate his position when Mei enters the interaction to help him clarify the nature of the category of odd-and-even. In Part 2, Mei shifts roles and acts as a dissenter, although she does so by appropriating Sean's perspective. Proceeding in this manner, she justifies her dissent in Part 3 by acting as a reasoner and generating a second example of odd-and-even. Her dissent from within Sean's viewpoint, it turns out, is mathematically generative, and the roles of accountable argumentation support this work.

Part 1: Clarifying. Sean, as the principal of this controversy about six, is engaged in an accountable argumentation sequence. He is standing at the board trying to prove his statement to his current dissenter, Tembe:

1	Sean:	Because, because see this, there's two *((drawing))* number two over here, put that there. Put this here. OO|OO|OO There's two, two, and two. And that would make six.
2	Mei:	I think I know what he's *say*ing.
3	Tembe:	Which is *even*, Sean.
4	Ball:	Mei?
5	Sean:	Yeah, and it could be (odd). *((Sean walks away from the board))*
6	Ball:	Could you stay there? People have some questions for you
7	Mei:	I think what he is saying is tha:::t. -h, it's almost, see/ *((Mei stands up in her seat))* I THINK what he's saying is that *((Sits back down, grabbing back of seat))* you have THREE *((holds up three fingers))* groups of TWO. *((holds up two fingers))* And three is an ODD number -h *((waving two fingers; rotates to face Ball))* so (.) SIX can be an odd number a::nd a even number.

Note that Ball maintains Sean at the center of the classroom (turn 6). Ball tells him to stay at the board, the physical center of the classroom discussion, and explains that people have questions for him, highlighting his corresponding interactional centrality. The expectations of the principal role are maintained in her request: Sean must continue to stand up — quite literally — for his position. After his unsuccessful attempt to explain his position to Tembe, he seeks to abandon his physical (and perhaps some of his interactional) centrality.

In turn 2, Mei steps in as a clarifier. Once her turn is ratified, she begins in turn 7 what O'Connor and Michaels (1996) call a *revoicing* move. Revoicing is "a particular kind of reuttering (oral or written) of a student's contribution -- by another participant in the discussion" (p. 71), which then opens up a slot in conversation for the original author of the statement to agree or disagree. In her role as clarifier, Mei does the first part of the

revoicing move: she reformulates Sean's statement about six, highlighting his focus on the fact that three, the number of groups of two, is an odd number. In describing the pattern, Mei's reformulation not only clarifies but starts to generalize Sean's statement.

Ball then finishes the revoicing move by opening up the slot to Sean for confirmation or disconfirmation. O'Connor and Michaels explain that it is this part of the revoicing move that "ultimately credits the contents of the *re*formulation to the student" (p. 71). Thus, Sean, whose talk has been animated and reformulated by Mei, maintains his participation status in the conversation as the *originator* of the statement, as he is positioned to confirm or disconfirm Mei's interpretation of his statement.

Part 2: Dissent. Mei then changes her interactional role. Ball asks her if she disagrees with Sean, designing her as a dissenter, a role that Mei readily assumes:

13	Mei:	Yeah, I disagree with that because -h *((Stands up facing Ball))* it's not acco::rding to like, -h/ *((Pushes chair under table))* Here. Can I show it on the board?
14	Ball:	*((pacing behind Mei's group's table))* Um hm *((nods)).*
15	Mei:	*((walking toward board, where Sean stands, leaning))* It's *not* according *((arriving at board, picking up chalk))* //to like how many *groups* it is.] *((pointing to Sean's diagram))*

Mei positions herself as a dissenter in relation to the reinterpreted statement about six. In turn 13, Mei ratifies the position of dissenter by disagreeing "with that" — Sean's position, not Sean himself. This is another strategy dissenters employ to manage the potentially personal feelings that come along with disagreement.

In assuming the role of dissenter, Mei immediately begins moving in to support her position. In turn 13, she gets out of her chair while beginning to state her opposition. She seeks Ball's permission to go to the board, where Sean has been positioned in turn 6. Moving and talking simultaneously, Mei states her point: "It's *not* according to like how many groups it is." By naming

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what she disagrees with, she further generalizes Sean's statement about six to describe a characteristic (the number of groups of two) that could also be found in other even numbers. Mei thus voices her dissent by arguing on more general grounds.

Importantly, Mei's work as clarifier helped open a path for her dissent. Recall the previous disagreement between Sean and Sheena in Episode 1, where Sean's dissent employed a different feature of even numbers than that on which Sheena's position was based (pairs of like-numbers versus the alternating pattern on the number line). In contrast, Mei first establishes the grounds for Sean's thinking about six and then disagrees with him on his own terms, in what might be thought of as *empathic dissent*. That is, she projects herself into his perspective in order to understand him better and continues to argue from within that perspective.

Part 3: Generalization through a counterexample

16 Ball: //Riba, can you watch what Mei's doing?]

17 Mei: Let's say:: that I ha::ve/ (7)
Let's see.
IF YOU CALL *six*
((points to Sean's drawing of six))
an odd number, why don't =
((facing Sean))

18 Sean: *((under breath))* = Or it could be an even.
((Standing at board, legs crossed))

19 Mei: *((quietly, facing Sean))* Let's see (if I find). (3)

Let's say ten. One, two . . . *((draws))*
OOOOOOOOOO

And here are ten circles. (1)
((underlines circles in air with her hand))
-h. And the::n you wou::ld SPLIT them, let's say I wanted to split, spit them, split them by *twos*.
One, two/
OO|OO|OO|OO|OO

And look. One, two, three, four, five .
((taps chalk against each pair of circles))
THEN WHY DO YOU NOT CALL, -h, *ten* a, like-- a
((Facing Sean, putting chalk back in tray))

20 Sean: *((smiling))*
//I disagree with myself. I call ten (an odd and even)]

21 Mei: // an *odd* number]
A::ND an *even* number? or why don't you call *other*, like numbers an odd number or-- and an even number?

22 Sean: (3) I didn't think of that that way. Thank you for bringing it up, so:::
I say it's:: (1).
Five/ ten can be an odd and an even?
//(???) can be an odd and a leven, an even]

In Episode 1, Sean merged the roles of dissenter and questioner to substantiate his disagreement with Sheena. Here, Mei has moved through the role of clarifier to dissenter, and finally, to dissenter-reasoner. She applies her established understanding that, in his statement about six, Sean is attending to *the number of groups of two* to generate a second example of an odd-and-even number, the case of ten (turn 19). Not only has she taken his perspective on the category of odd-and-even, but her example uses the same inscription that he used in his interaction with Tembe, underscoring her strategy of appropriating his viewpoint in empathic dissent. From her central position at the board, she generates a second instance of an odd-and-even number using the same "cookie" representation of numbers. By reasoning from within his perspective, accountable argumentation is sustained in structuring this interaction.

[Insert Figure 2 about here]

The case of ten provides an even more significant move toward generalization: two examples of numbers with the same property (they can be partitioned into an odd number of pairs) open the possibility for a mathematically defined class of numbers. In her role as reasoner, Mei clearly generates the case of ten; however, because she is reasoning in service of her dissent, she does not interactionally take ownership of the example of ten. For her purposes, it is a strategic attempt to find a counterexample: a number that is known to be even but also has an odd number of groups of two. Because of her purposes in generating this example, she continues to position Sean as the originator of the example of ten in turns 19 and 21 ("Why do you not call *ten* a - like a" "an *odd* number A::ND a *even* number?"). Although she has generated the second case of an odd-and-even, her statement continues in the second person (you) and thus maintains the category and the example as his. In doing so, she completes the revoicing move by then opening the slot for Sean to confirm or disconfirm her reformulation. After a

three second pause in turn 22, Sean politely (if a little falteringly) revises his position to include both ten and six in the category of odd-and-even numbers.

Sean is active in the coconstruction of the odd-and-evens in this segment. From his position as principal, he asserts the boundaries of his category for six in turn 18 by correcting Mei's restatement of his claim. By insisting that he is not claiming six is *odd*, but rather *odd-and-even*, he maintains the category of numbers under construction. Mei takes up the refined category in turn 21 with an emphatic "A::ND" linking the stressed words *odd* and *even*, underscoring that they are indeed talking about the same kind of number.

Sean thanks Mei for bringing up the case of ten (turn 22). Accountable argumentation, with its expectation of modifying positions in light of convincing evidence, provides an interactional resource for Sean to "revise" his position without losing face (Renkema, 1993). Sean concedes to disagree with himself (turn 20), revising his position, and allows ten as a second case of an odd-and-even. Since he is still positioned to confirm or disconfirm this second reformulation, he maintains his role as principal of the controversy, as well as originator of the odd-and-even category.

Accountable argumentation sequences are primarily managed by the students engaged in them. Nonetheless, Ball plays an important role in their execution. Although she is peripheral to the interaction, Ball does two kinds of work in this segment, one visible and one invisible. The visible work she does is to help Mei maintain the floor. By asking Riba to watch Mei, Ball invokes her teacher's authority to support Mei's authority to speak. The invisible work that Ball does in this segment is one of non-intervention, related to the normative measured pace that supports accountable argumentation. Mei takes several pauses — one which lasts for seven seconds (turn 17) — while she is struggling to find her example of ten. Ball's silence and nonintervention allow Mei to do that challenging mathematical work.

It is interesting to contrast Ball's choice of nonintervention during this seven-second pause to her decision to intervene after Sheena's two-second pause in Episode 1. Recall that two seconds after Sheena has put the yardstick back in the tray, Ball steps in to close the argumentation sequence. During the pause in Episode 1, Sheena has signaled her readiness to relinquish her role as principal, while during Mei's seven-second pause, no such signal is

provided. In fact, Ball has just signaled Riba to "watch what Mei is doing," recognizing that Mei is hard at work and should not be interrupted. This contrast highlights the subtle contextual cues that Ball relies on to support her students during accountable argumentation sequences.

Discussion of Episode 2: Through her roles as clarifier and then as dissenter-reasoner, Mei helps to move Sean's statement about six from a particular claim about six to a general class of numbers. Although further work by other class participants generates other examples of odd-and-evens, eventually mapping them onto a pattern on the number line pattern, Mei's strategy of empathic dissent contributes to the mathematization of Sean's original statement. Empathic dissent, in contradistinction to the cross-representational talk of Episode 1, proves to be mathematically productive here and later in the class session. Of course, Sean and Ball also contribute to the coconstruction of the concept during this episode, but Mei has done significant mathematical work.

The transformations of the statement about six can be summarized in Figure 3 below. In this figure, some key statements that highlight the construction of generalization from Episode 2 are placed in chronological order. Mei first clarifies Sean's original statement, in the top box. Her summary, in the second box, highlights his focus on the odd number of groups of two. The third box contains one of Mei's dissenting statements. In objecting, she names the source of the disagreement as "the number of groups," a more general description of a "type" of number. The fourth box contains the second case of odd-and-even numbers, generated as a counterexample by Mei from her position as a dissenter.

[Insert Figure 3 about here]

DISCUSSION AND QUESTIONS

Let us now return to the questions that introduced this study: How can classroom discourse be organized to support mathematical disagreements that (a) are intellectually productive, and (b) minimize social discomfort? This analysis of the interactional organization of Deborah Ball's January 19, 1990 class provides one answer to these questions.

In this classroom, accountable argumentation brings the often hidden practices of mathematical reasoning into the visible world of classroom interactions. However, accountable argumentation entails something more subtle than an outright modeling of mathematical thinking. The social risks commonly associated with disagreement manage to be mitigated by other elements of its structure.

How does accountable argumentation make mathematical thinking a visible activity? First, the structure provided by accountable argumentation helps make disagreements intellectually productive in several ways: by providing support for "thinking activities", by supporting deep engagement with specific ideas, and by supporting the learning and creating of new mathematics. Two norms of accountable argumentation, in particular, support thinking activities in the classroom: the use of terms from the mathematical and academic registers, such as "proving" and "conjecturing", and the norm of a slow and measured pace of discussions that permit such activity to take place.

During the class session, these thinking activities are focused on a particular set of ideas. Accountable argumentation supports engagement with specific ideas, particularly through the expectations that (a) students attend to whole class discussions and (b) students take a justified position in a discussion which they will act on or defend. In addition, once they are engaged in a disagreement, the stakes for engagement increase. Dissenters are obliged to ask questions or otherwise substantiate their position to their peers. Principals, on the other hand, must articulate their thinking to the whole class.

Effectively, these thinking activities and the engagement in particular ideas support both the learning and creation of mathematics. When Sheena has to articulate her position on zero, she transforms a vague statement to her teacher into a well-reasoned position to the whole class. This change can be viewed as increased competence and is perhaps indicative of her learning. Likewise, Sean "revises" his thinking about six as a unique "odd-and-even" number, recognizing that Mei's example of ten is also "odd-and-even." He incorporates an example that had not been previously part of his thinking; he, too, has learned something new. Perhaps most striking, through the class' discussion, the particular observation about one number becomes the first of many examples of a general class of numbers — a counterexample is

transformed into a second example through the process of argumentation, mirroring a way in which mathematics is created by mathematicians (Lakatos, 1976).

How do Ball's students tolerate the tension that is usually associated with disagreement in our culture? It turns out that accountable argumentation provides subtle resources to mediate these tensions. The normalization of disagreements seems to play a significant role in alleviating potential risks. The students appear to know the norms, expectations, language, and roles through which to argue. Ball, through her talk, underscores disagreement as a collective, not personally threatening, activity through her use of the first person plural ("we're probably not going to settle this").

Additionally, the discourse of accountable argumentation consistently distinguishes it from potentially threatening, personal disputes. By couching disagreements in academic terms, language provides not only a marker but a resource that allows students to challenge each other ("Prove it to us!"), make uncertain guesses ("conjecture"), and change their minds ("revising"), while avoiding the corresponding social costs of being deemed aggressive, foolish, or cowardly. Also, the deliberate civility of the exchanges ("please", "thank you") serves as a reminder that these arguments are not meant to be personal. In addition, students sometimes deliberately disagree with *positions* ("what he said") not *people*, again distancing the intellectual from the personal. Finally, disagreements are often initiated in the third person, changing to second person only after the principals have ratified their position.

This analysis raises questions that merit further investigation. For example, how does a complex participant structure such as accountable argumentation develop in a classroom community? By taking a longitudinal perspective on Ball's video data, one could trace its evolution over time. Such an analysis would provide an important guide for other teachers trying to cultivate productive mathematical discourse in their classrooms. Investigation of a second question would also support such endeavors: what is the teacher's role in supporting productive mathematical disagreements in the classroom? In the two episodes analyzed in this paper, Ball does subtle work in deciding when to pursue or curtail students' disagreements. Such teaching requires an acceptance of ambiguity in the classroom, something that is not comfortable for all teachers (Doyle, 1988). Finally, could there be other

forms of accountable argumentation? Which of the norms, expectations, and roles are necessary for it to function productively in different contexts? One could imagine that certain modifications would be required for different teachers, different students, or even different subject domains.

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TABLES AND FIGURES

/	Self-interruption
=	No gap between utterances
(.), (5)	Very slight pause, five second pause
//,]	Beginning of overlapping utterances, end of overlapping utterances
,	Low rise in intonation
?	High rise in intonation
::	Marks lengthened syllable, each : equals one "beat"
.	Low fall in intonation
italics	Marks stress
capital letters	Increased volume
-h, h, (h)	in-breath, out-breath, laughter
(??), (cow)	Unclear reading, tentative reading
(( ))	Marks other voice qualities or actions

Table 1: A summary of transcript conventions, adapted from Ochs, 1979.

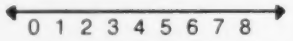
Name	Representation	Highlights:
Number line		alternating pattern of even and odd
"Cookies"	O O O O O	Groups or pairs; splitting in half
Hash marks	| | | | | | | | |	Groups or pairs

Table 2. Inscriptions used during whole class discussion on January 19, 1990. Each representation highlights a different feature of even and odd numbers.

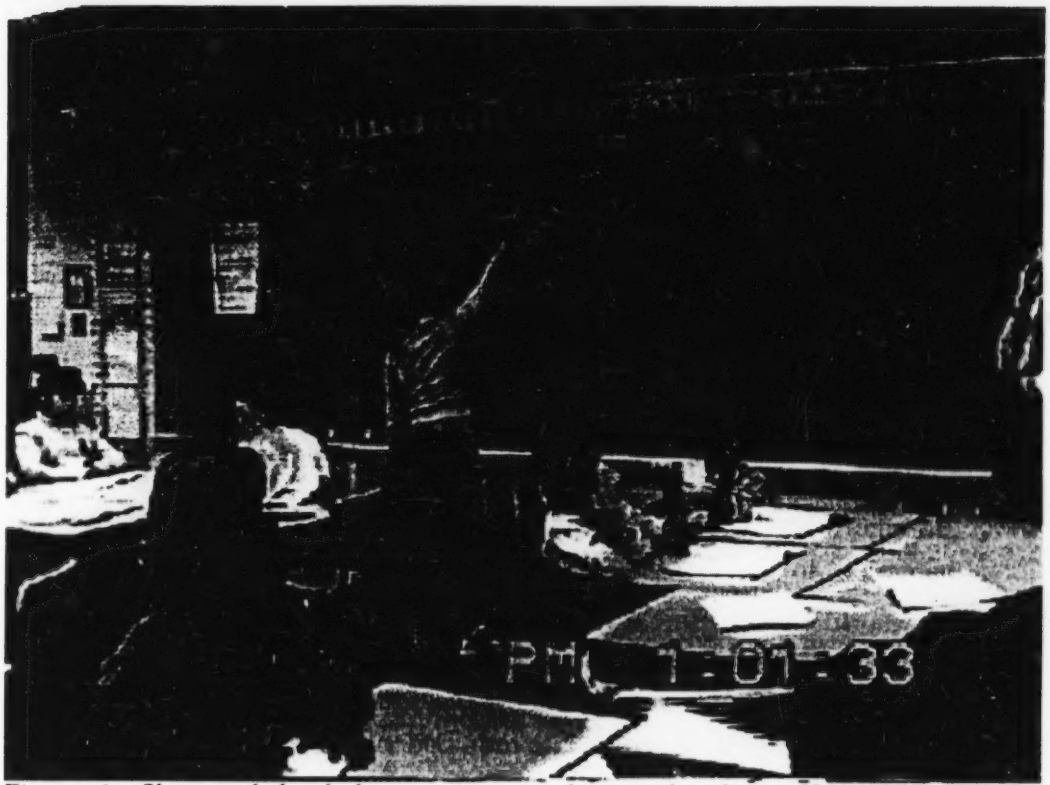


Figure 1. Sheena defends her position at the number line. (Source: videotape of Deborah Ball's class, January 19, 1990, M.A.T.H. Project.)

Figure 2. Sean and Mei disagreeing about six. Note Mei's drawing of ten, below Sean's drawing of six, on the chalkboard. (Source: videotape of Deborah Ball's class, January 19, 1990, M.A.T.H. Project.)

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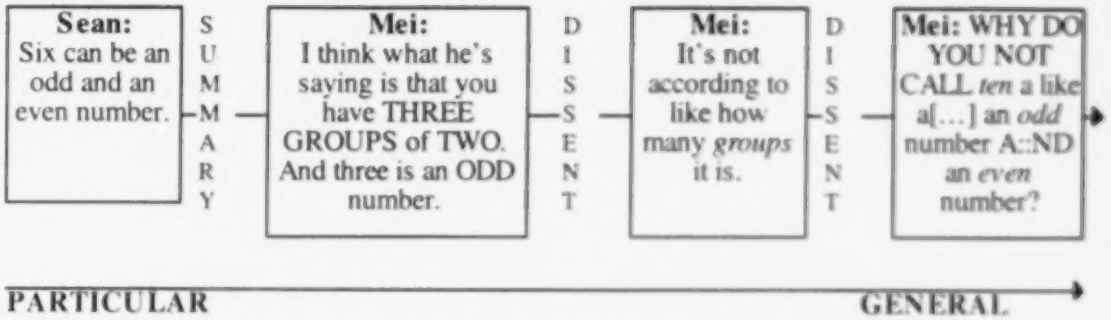

Figure 3. A map of the movement from the particular to the general through accountable argumentation in Episode 2.

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